

Facts about Integration by Parts for Indefinite Integrals	Explanation
$\int u dv = uv - \int v du$	The common Integration by Parts Formula for Indefinite Integrals.
<p>The LIATE Scale</p> <p>L - Logarithm Function  I - Inverse Trigonometric Function <math>\uparrow u</math>  A - Algebraic Function (polynomials)  T - Trigonometric Function <math>\downarrow dv</math>  E - Exponential Function (<math>e^x</math> or <math>10^x</math>)</p>	<p>Use this LIATE scale to pick your <math>u</math> and <math>dv</math>. Pick your <math>u</math> to be something higher on this scale and <math>dv</math> to be something lower on this scale.</p> <p>This is just a guideline; there might be functions where you might not want to use the LIATE scale.</p>
How to compute $du$ where $u = f(x)$ ?	$du = f'(x) dx$
How to compute $v$ where $dv = g'(x) dx$ ?	$v = \int dv = \int g'(x) dx = g(x)$

1. Evaluate  $\int xe^x dx$  through the following parts.

(a) Use the LIATE scale to assign your  $u$  and  $dv$ .

**Solution:**

$$u = x \quad dv = e^x dx$$

(b) Find  $du$  (the differential of  $u$ ) and  $v$  (antiderivative of  $dv$ ).

**Solution:**

$$du = dx \quad v = \int e^x dx = e^x$$

(c) Set up the integration by parts formula and find an antiderivative for the integral.

**Solution:**

$$\begin{aligned} \int xe^x dx &= xe^x - \int e^x dx \\ &= xe^x - e^x + C \end{aligned}$$

2. Evaluate  $\int x^2 \ln(x) dx$  through the following parts.

(a) Use the LIATE scale to assign your  $u$  and  $dv$ .

**Solution:**

$$u = \ln(x) \quad dv = x^2 dx$$

(b) Find  $du$  (the differential of  $u$ ) and  $v$  (antiderivative of  $dv$ ).

**Solution:**

$$du = \frac{1}{x} dx \quad v = \frac{x^3}{3}$$

(c) Set up the integration by parts formula and find an antiderivative for the integral.

**Solution:**

$$\begin{aligned} \int x^2 \ln(x) dx &= \frac{x^3}{3} \ln(x) - \int \frac{x^3}{3} \frac{1}{x} dx \\ &= \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C \end{aligned}$$

3. Evaluate  $\int e^x \sin(x) dx$  (You will have to use integration by parts twice).

**Solution:**

We first find

$$\begin{aligned} u &= \sin(x), & v &= e^x, \\ du &= \cos(x), & dv &= e^x dx. \end{aligned}$$

This leaves us with a new integral

$$\int e^x \sin(x) dx = e^x \sin(x) - \int e^x \cos(x) dx.$$

Unfortunately, we still don't know how to solve the integral we've been left with, but let's just integrate by parts one more time. Let

$$\begin{aligned} u &= \cos(x), & v &= e^x, \\ du &= -\sin(x), & dv &= e^x dx, \end{aligned}$$

and we find

$$\int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) dx.$$

But wait a minute. We can just add the last term over to find

$$2 \int e^x \sin(x) dx = e^x \sin(x) - e^x \cos(x).$$

This is just twice what we originally wanted to find! So, we have

$$\int e^x \sin(x) dx = \frac{e^x \sin(x) - e^x \cos(x)}{2} + C.$$

4. Evaluate  $\int_0^1 \arcsin(x) dx$ .

**Solution:**

There is not much to go on here, but we can begin by choosing

$$\begin{aligned}u &= \arcsin(x), & v &= x, \\du &= \frac{1}{\sqrt{1-x^2}} dx, & dv &= 1 dx.\end{aligned}$$

Now we integrate by parts to find

$$\int \arcsin(x) dx = x \arcsin(x) - \int \frac{x}{\sqrt{1-x^2}} dx.$$

With this new integral, we can make the substitution  $w = 1 - x^2$  to find

$$\int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2},$$

and so,

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{1-x^2}.$$

The original problem was a definite integral from 0 to 1, so we evaluate as

$$\int_0^1 \arcsin(x) dx = x \arcsin(x) + \sqrt{1-x^2} \Big|_0^1 = \frac{\pi}{2} - 1.$$

5. Some additional practice.

1.  $\int 4x \cos(2 - 3x) dx$

2.  $\int_6^0 (2 + 5x)e^{x/3} dx$

3.  $\int x^2 \cos(3x) dx$

4.  $\int t^7 \sin(2t^4) dt$